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The Influence of Nonmetallic Inclusions on the Mechanical Properties of Steel: A Review

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The literature regarding the influence of nonmetallic inclusions on the mechanical properties of steel is reviewed, with critical comments on various studies. A brief discussion of inclusion rating methods and a synopsis of the effects of applied stress on inclusions in an isotropic, elastic matrix are presented. The parameters considered are tensile strength, impact strength, reduction of area, fatigue properties and fracture toughness. It is concluded that in many applications, the type of inclusions are more important than the total content and as matrix strength increases, the notch effect of inclusions becomes more significant. Also, mechanical properties can be influenced by any one or a combination of the following inclusion parameters; shape, size, quantity, interspacing, distribution, orientation, interfacial strength, and physical properties relative to the matrix.

1. Introduction

Two kinds of nonmetallic inclusions are generally recognised in steel; those which are entrapped in the steel inadvertently and those which separate from it because of a change in temperature or composition [1]. The first type originates almost exclusively from particles of matter that are occluded in the steel while it is in the molten state. The second is produced either in the molten or in the solid state by separation from the steel of oxide, sulphide, nitride or other nonmetallic compounds in the form of droplets or particles, when these compounds are produced in such amounts that their solubility in the metal is exceeded. Since the latter are products of reactions within the steel, they are normal constituents of it. Because ordinary manufacturing processes cannot entirely rid the steel of such inclusions, it is desirable to control their kind and amount within such limits that the steel is relatively free from those inclusions which are considered most injurious. Typical steel making parameters that influence the resulting inclusions are; time of carbon boil, refractory composition, deoxidation practice, composition of the deoxidising alloy additions, tapping and teeming operations [2]. Nonmetallic inclusions typical of steels are illustrated in figs. 1 and 2.

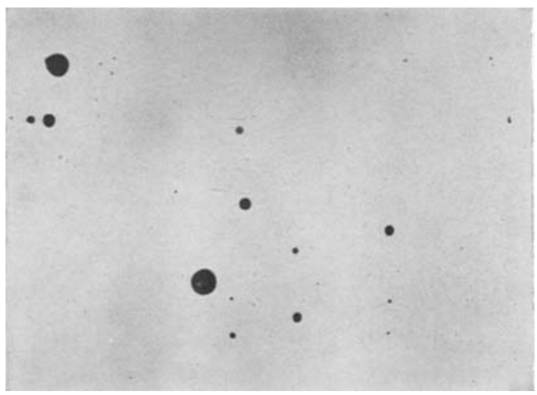

Figure 1 Typical oxide inclusions in low alloy steels (× 100).

2. Inclusion Measurement

Typical rating systems encountered in this survey are; ASTM-E45 "A" (comparative), Fairey inclusion count (comparative), lineal analysis (direct measurement), SAE method (combination) and ultrasonic detection methods. These methods are representaitve of rating systems in general use throughout the steel and fabricating industries. Although the procedure for each method varies somewhat, the final results depend heavily on the individual investigator's skill.

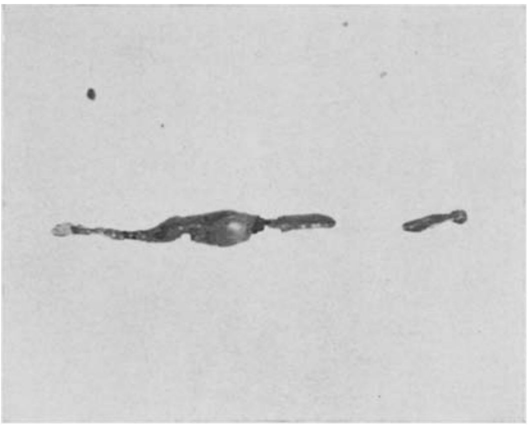

Figure 2 Complex inclusion of Fe-Mn-Si (dark portion) associated with sulphides (light portion) (× 500).

Therefore, the requirement of consistency, essential to all experimentation, is usually not satisfied. Since any approach to this problem is confronted with formidable complications, each rating system with its inherent limitations must be considered with respect to the information required.

3. Effects on Stress

3.1. Inclusion Stress

In the elastic range, the principal effects of inclusions are to increase stress magnitudes in local areas due to stress concentration and to alter or eliminate uniaxiality of the stress field [4]. Stress concentrations within inclusions of specific shapes have been determined by Edwards [5] and Goodier [6]. Their results are used in fig. 3 to compare the maximum tensile stresses in a single spherical inclusion and a single cylindrical inclusion of axis perpendicular to the stress direction, both being subjected to a uniform uniaxial tension at infinity. The stress concentration is maximum for spherical inclusions of a perfectly rigid material ($E_1/E_2 = \infty$, subscripts 1 and 2 refer to inclusion and matrix resp.) attaining $\sigma_1 = 1.92\,\sigma_2$ if Poisson's ratio of the inclusion is taken as 0.33 and it occurs along the polar axis of the sphere parallel to the stress direction. If $E_1/E_2 = 0$ the problem reduces to that of a cavity. Stress concentrations may reach much higher values for elongated particles with longitudinal dimensions perpendicular to the direction of the applied stress.

3.2. Interface Stress

The surface forces at the interface are the same for the matrix and the particle materials. The magnitude and location of the maximum stresses are determined by the shape of the particle and the ratio of the elastic moduli of the two materials. The radial and tangential stresses at the interface are shown in fig. 4, at poles 0, 45 and 90 degrees from the stress direction. The maximum tensile stress occurs at the 90° pole of the cavity and at the 0° pole of the inclusion.

3.3. Matrix Stress

The stresses in the adjacent matrix material are indicated in fig. 5. The perturbation of the elastic stress field diminishes rapidly as a function of distance from the interface.

4. Results of Investigations

4.1. Strength

Ultrasonic inspection was employed for the evaluation of inclusion content on 197 Kg/mm² (UTS) 4340 steel [7]. The results of the tensile data (averages of six specimens) showed that the effect of inclusion content on ultimate tensile

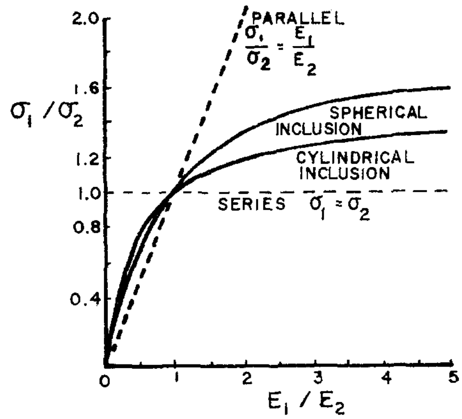

Figure 3 Stress concentration at inclusion [4].

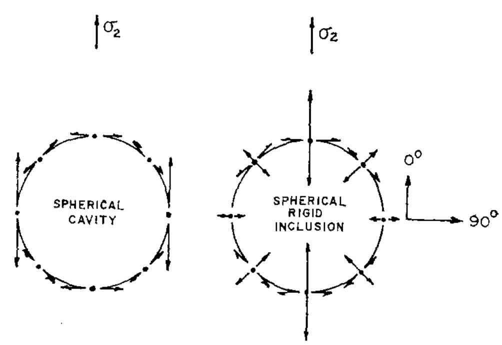

Figure 4 Interface stresses [4].

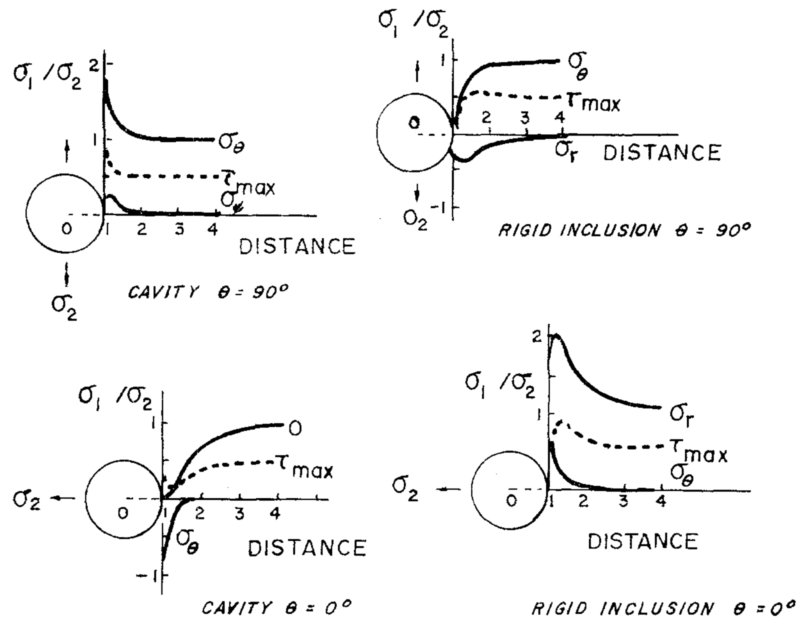

Figure 5 Matrix stresses (σ_r= radial stress, $\sigma\theta$, $\sigma\psi$= tangential stresses, τ_{max} = max. shear stress) [4].

strength was negligible (fig. 6). The yield strength, however, was reported to increase with increasing ultrasonic inclusion category. The authors attributed this to a higher degree of triaxial stress present in the "dirtier" (higher category) specimens with a greater constraint of lateral contraction. But considering that this change of approximately 7 kg/mm^2 over the entire inclusion category range amounts to only about 5% of the average yield strength and that variations of this order have been noted in this material as the yield strength exceeds 133 kg/mm^2 [8], this effect could be misleading. Also, it should be noted that the ultrasonic inclusion categories did not correlate with metallographic inspection of the specimens (correlation coefficient r = 0.54 for their data).

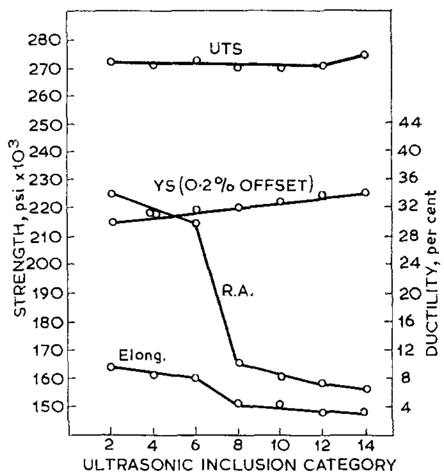

Figure 6 Transverse tensile properties of 4340 steel versus ultrasonic inclusion category [7].

4.2. Impact Strength

The effects of nonmetallic inclusions on the impact strength of low carbon steel containing about 3% Ni, and medium carbon steel containing 0.9% Cr and 0.13% V, were investigated by Kinzel and Crafts [9]. Generally a decrease in impact energy with increased inclusion content (number per mm^2) was reported. Considerable scatter was evident in their data so that a trend rather than a direct relationship was indicated. This trend was more pronounced in the transverse tests. Moreover, they stated that for the softer steel the loss in the "dirtiest" material was approximately 25% (transverse tests), whereas in the harder steel, the maximum loss amounts to about 50%, indicating that the loss of impact strength due to inclusions increases as the hardness increases. In a follow-up study, the same investigators conducted similar tests on samples of SAE 2315 steel [10]. The specimens were divided into three groups with respect to cleanliness. Although a trend was indicated by their data only one important conclusion was drawn; "that inclusion content alone is not a satisfactory criterion of the dynamic properties of steel and may even be misleading".

4.3. Reduction of Area

Approximately 100 high temperature tensile tests were conducted [11] on 0.30 C, low alloy steel. This test appeared to be particularly severe in terms of activating inclusions to form surface or subsurface defects. Holes which resulted from either fracture within nonmetallics or failure at inclusion-matrix interface, elongated in the tensile axis direction.

Reduction of area was studied in several heats of 4340 steel taken from 40 mm gun tubes [12]. It was proposed that the same factors which cause anisotropy in the RA also cause anisotropy in fatigue properties, since both are controlled by crack initiation and therefore depend on local rather than average characteristics in the steel. The total inclusion content for all the tubes from each heat was nearly the same and at a fairly low value. Since no fatigue tests were conducted, it is speculative to assume the inclusion effects on transverse ductility will apply to transverse fatigue properties. But their results demonstrated that the type of inclusion was more important than the total amount of inclusions, in determining the transverse ductility (RAT) of this steel.

Welchner and Hildorf [13] attempted to relate the inclusion content and transverse ductility in a Cr-Ni-Mo gunsteel. The relative quantity of each of four inclusion types (spherical, irregular particles, discontinuous, and continuous stringers) was determined by estimating the percentage of each present in the specimen (× 100), the total of the four types being 100%. They concluded that:

(*a*) There is a definite correlation between the average inclusion content and average RAT. Average RAT was lowered about 10% for each unit of inclusion rating increase.

(*b*) A 10% increase in stringer-type nonmetallics decreases the average RAT about 3%, independent of the total inclusion content.

(*c*) The type of inclusion has a greater effect than the total amount of nonmetallics, on the transverse ductility of this steel.

The investigators pointed out that their first conclusion above did not regard the type and/or distribution of these particles. It should also be noted that variations in RAT ranged from 4 to 46% for a single gun tube, as reported in the study mentioned previously [8], so that changes of 3% RA are not very significant.

A study previously described [7], also investigated the influence of inclusions on ductility. In 4340 steel, reduction of area proved to be the most sensitive respondent to material cleanliness measured ultrasonically. A difference of 28% RA occurred between the low and high category transverse specimens shown previously in fig. 6.

An investigation conducted on six 175 mm M113 gun tubes utilised an ultrasonic detection system to assess the non-metallic inclusion content [14]. Ultrasonic severity ratings were obtained for longitudinal tube sections. These ratings represent weighted numbers of counts per unit volume of material inspected. Although no correlation was reported between the results of this method and the metallurgical inspection, the investigators contended that the severity rating increased as the cleanliness decreased. The data plotted in fig. 7 shows the average reduction of area as a function of ultrasonic severity rating. For two tubes only two specimens were tested and from lack of data it must be assumed that the other points represent an average of two tests also. Therefore, in view of the nature of the inclusion measurement and number of tests conducted, the reported relationships are probably unjustified.

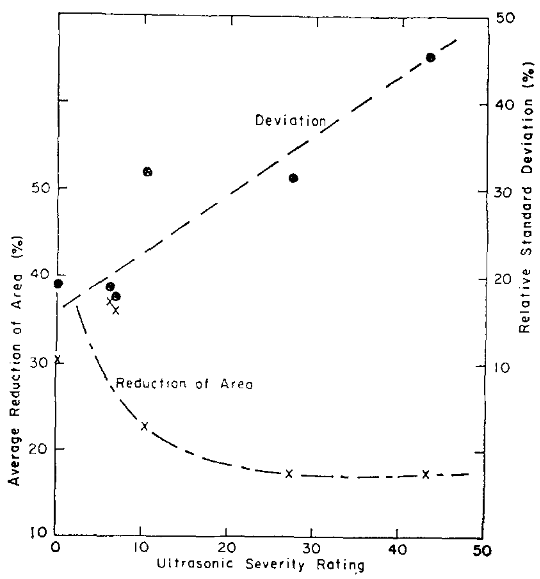

Figure 7 Relation of reduction of area statistics with ultrasonic severity rating [13].

4.4. Fatigue Properties

By fractographic analysis, Pelloux [15] showed the influence of constituent particles on fatigue strength in two heats of 7178 aluminium alloy which differed only in impurity content. This study has been included because of the similarity in fatigue crack propagation between this material and steel, plus the manner in which it was reported. The volume fraction was estimated by a point count technique to be:

Alloy	*Volume Fraction* (%)	*Estimated Error* (%)
A	0.38	± 0.078
B	4.78	± 0.36

Figs. 8a and b are representative of the fracture features in alloy A and B respectively. The microscopic growth rate (fatigue striations) depict the local rate of crack propagation through the matrix. The larger macroscopic rates for the two alloys (fig. 9) demonstrate the effect of the inclusions in the "dirty" alloy.

Fatigue crack propagation was investigated in hot rolled, normalised, steel plate [16]. No attempt was made to establish a quantitative relationship between inclusions and fatigue properties, but the data indicated that the fracture mechanism which had the greatest control over the observed macroscopic growth rate was local fracture at inclusion-matrix interfaces. (As described by Pelloux, the macroscopic growth rate was considered to be the summation of several fracture mechanisms, the most important being striation formation and local fracture of brittle microconstituents [inclusion-matrix fracture].) Striation spacing was seen to be independent of orientation, suggesting that fracture of inclusions was responsible for the crack growth rate anisotropy. Completion of the crack initiation stage, very early in terms of the total fatigue life, has been reported in conjunction with gun tubes [17]. Therefore the cyclic crack propagation and critical crack size are of primary importance in determining this life. In the light of the analyses by Pelloux and Heiser, the influence of inclusions on low cycle fatigue applications may be very substantial.

The effects of nonmetallic inclusions on crack propagation were studied in the low alloy steel 45-L [18]. The inclusions obtained were identified as:

Inclusion Type	*Description*
1	Globular (Mn-Si or Al-Si)
2	Sulphide films
3	Acicular sulphides

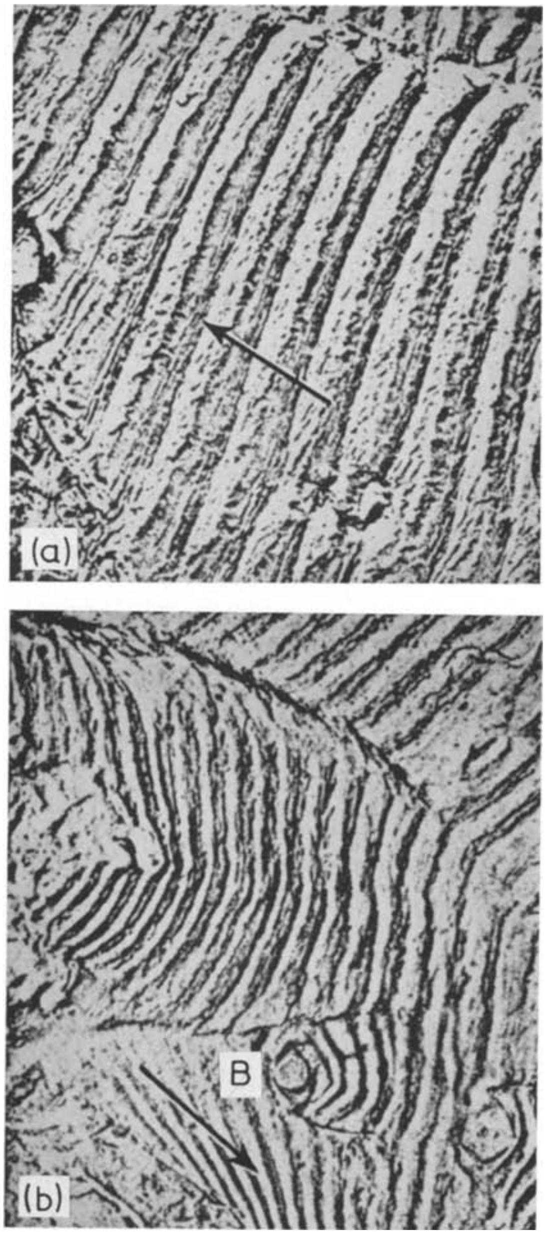

Figure 8 Fracture surfaces of two 7178 aluminium alloys. 8a-Alloy A (× 4000), 8b, Alloy B (× 2000).

Qualitative metallographic examination of specimens led the investigators to conclude that inclusions play a significant part in the nucleation and propagation of cracks, with type 2 situated at grain boundaries being most detrimental. Crack formation at the tips of type 3 was noted and in steels with types 2 and 3, crack propagation took place preferentially through these

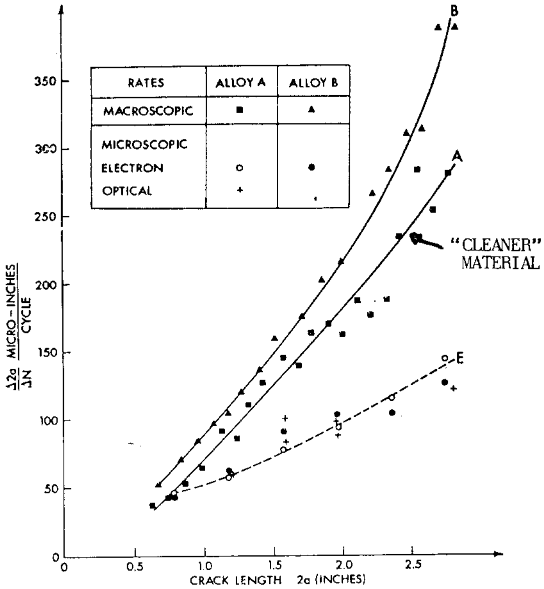

Figure 9 Growth rates versus crack length for 7178 aluminium alloys. *E* measured from striation spacings [14].

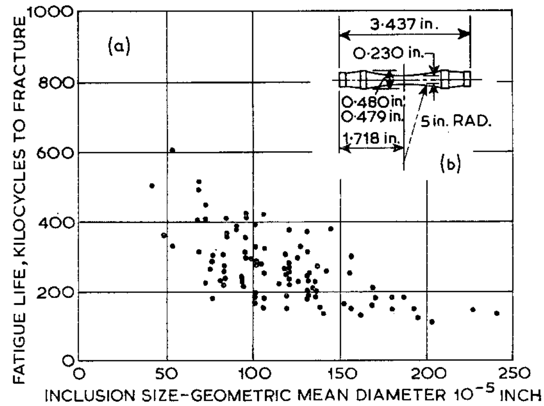

Figure 10 Fatigue life versus inclusion size [18].

inclusions. It appeared that type 1 inclusions were least capable of promoting crack propagation. Despite the statements made by these investigators, it should be noted that the inclusions were not quantitatively rated and the amount of data presented in the report is not representative of a comprehensive study.

An attempt to relate nonmetallic inclusions to the fatigue properties of steel was made using SAE 4340 and SAE 52100 [19]. The inclusions were of the spheroidal type so the geometric mean diameter of the exposed cross-section was adopted as a size parameter. It was concluded that at a given UTS, the fatigue lives of these high strength steels, show an inverse dependence on the size of inclusions (fig. 10). Although the data in this figure shows a trend, the scatter indicates that the degree of this dependence is vague. Cummings *et al* later reported an improved correlation on the 4340 steel using a "least-squares" method [20]. The equation was not given because further studies showed that there is no simple, general, mathematical relationship applicable to all stress levels.

The fatigue properties of two commercial heats of 4340 steel (SAE inclusion rating O-O-A) were investigated [21]. The overall evidence pointed to fatigue failures that were initiated by inclusions whenever the long axis of the inclusion was parallel to a direction of maximum shear stress or normal to a direction of principal tensile stress. The origin of failures in bend specimens (taken parallel to the working direction) could not be traced to inclusions. It was concluded that the transverse fatigue limit of forgings made by conventional steel making practice is less than the longitudinal fatigue limit because of the presence of elongated inclusions.

Fatigue strength reduction factors, which indicate the ratio of fatigue strength that the specimens might have at 10^7 cycles if there were no inclusions, to the strength the specimens actually had with inclusions of various sizes present, were estimated for SAE 4340 and 4350 steels in the 100 to 210 kg/mm^2 range [22]. The dominant inclusions were spheroidal silicates. Fracture surfaces were examined at $\times$ 400 in polarised light and the inclusions nucleating the failures (single nucleus fractures) were measured with a micrometer eyepiece. The inclusions which nucleated fatigue failures ranged in size from 0.0254 to 0.0762 mm in diameter. The fatigue strength of specimens having inclusion diameters in the upper part of this range, were found to be substantially lower than those having smaller diameter inclusions. However, the total content and distribution of inclusions were not mentioned, so that the role of the smaller inclusions in the fatigue failure of the "large inclusion" specimens is unknown.

The influence of nonmetallic inclusions on the fatigue properties of ultra-high strength steels was investigated by Atkinson [23]. The inclusions observed, sulphides, silicates and complex types, were rated by the Fairey method. The data indicated a trend of decreasing fatigue life with increasing inclusion count, and although it was concluded that a correlation existed, considerable scatter was evident in this data. In view of the results, the investigator proposed that the Fairey inclusion count measures some relevant quality of inclusions, which appears to be a function of stress concentration and decrease in load bearing area due to the inclusions.

The effect of metallurgical variables on the fatigue properties of AISI 4340 steel in the 183-220 kg/mm² range has been studied [24]. Inclusion concentration was reported in terms of the number per square inch. Prominent types of inclusions in each size range were also evaluated in terms of length to width ratios (without considering orientation). Analysis of the size of the largest inclusions present indicated that an increasing ratio of endurance limit/UTS was associated with a decreasing inclusion width. Although data from five of these steels displayed a trend, three fell considerably outside any possible correlation. Also, a difference was noted between measurements of the size of apparent fracture nucleating inclusions and those measured *in situ*. This was interpreted to mean that the nucleating site was not entirely an inclusion but, a manifestation of the residual stress field surrounding it. In general, however, there were insufficient data at a given stress level to show any good relationship and the authors themselves stated that no satisfactory conclusions could be drawn from these results.

The investigation by Carter *et al* [7] on 4340 steel, concluded that fatigue response of unidirectionally loaded and reverse loaded specimens showed a dependence on inclusion severity in both the longitudinal and transverse directions. They showed a trend of increasing failure rate for increasing inclusion category. But these trends are not consistent in their data and could be attributed to their measurement techniques. In a crack propagation study currently being conducted by the present author [25] on cast 4330 steel plate (113 kg/mm² - yield strength) the role of nonmetallic inclusions in the fracture process has been observed by scanning electron fractography (figs. 11 and 12). The separation of inclusion-matrix interface is evident from the cavities surrounding the particles on the fracture surface. Fig. 12 exemplifies the role inclusions play in hole-coalescence (McClintock–MIT). It is also of interest to note the enlarged cavities around the inclusions which agrees with the difference in size of apparent fracture nucleating inclusions and those measured *in situ* as previously reported [24].

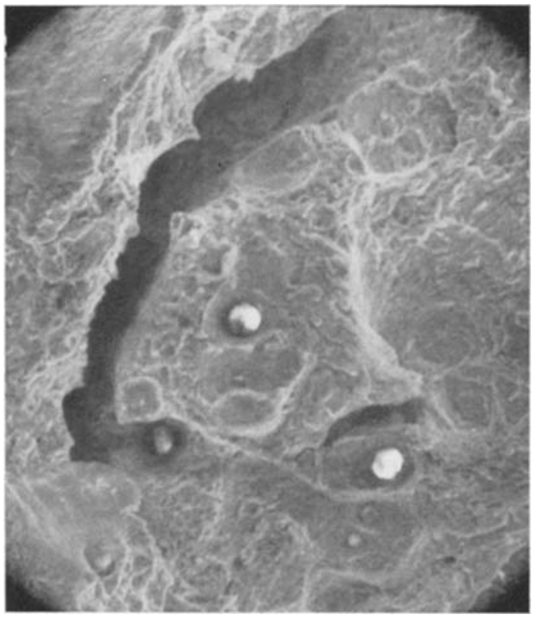

Figure 11 (Ca-Mn-Si) Oxide inclusions on fracture surface zero degree tilt (× 500).

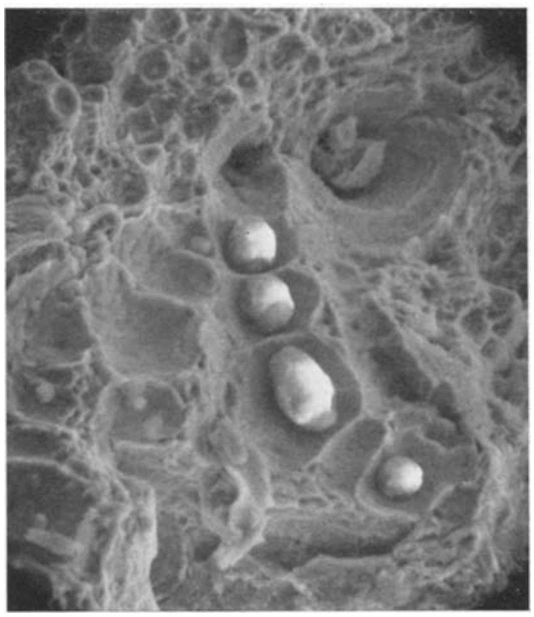

Figure 12 Oxide inclusions on fracture surface, zero degree tilt (large inclusion approximately 15 microns) (× 1000).

4.5. Fracture Toughness

English [26] presented an analysis of the influence of mechanical fibring (preferred orientation having a non-crystallographic origin) on fracture modes. Such fibring arises because inclusions, chemical segregation and 2nd phases are modified and aligned by the deformation accompanying mechanical working. Considering the ductile fracture mode, at least two factors are seen to have a role in producing anisotropy of fracture in fibred materials. First, the relationship of weak interfaces oriented with respect to the tensile stresses, governs largely the point at which internal fissuring begins and whether they are detrimental or beneficial. Second, there will usually be differences in accumulated plastic strain (from working) between the inclusion and the matrix. This strain leads to anisotropy in the apparent spacing to diameter ratio. This ratio is in turn a principal factor in governing the fracture strain once the inclusions have been separated from the matrix (hole coalescence). In the brittle mode, fibring introduces weak internal interfaces which may in appropriate orientations, reduce triaxiality locally or deflect a propagating crack, (fig. 13) and may therefore be beneficial [27].

Figure 13 Schematic of crack propagation in presence of weak interface (delamination) [26].

The role of inclusions in ductile fracture was examined [28] by considering the stresses which developed around inclusions and by microscopic observation of the initiation and growth of cracks which led to ductile rupture. Fig. 14 shows an example of such a surface where cavities formed at inclusions. Three different stages were proposed for this type of ductile rupture; (i) formation of cracks at inclusions, (ii) growth of the cracks, (iii) internal striction leading to failure. The elongation to rupture was calculated and it was shown that for a given material, it depends primarily on the volume fraction of the particles.

An attempt was also made by Carter *et al* [7] to determine the effects of inclusions on fracture toughness. Plane strain fracture toughness results, obtained with modified ASTM STP 410 slow bend bars, indicated very little, if any, dependence on material cleanliness. The scatter in these data is evident in fig. 15.

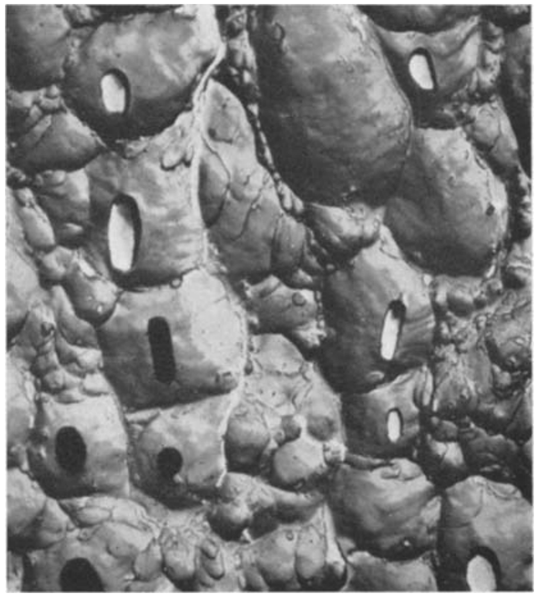

Figure 14 Fractograph of ductile rupture in a low carbon, 1.2% Mn Steel showing cavities formed by inclusions [16] (× 6000).

Recent work [29] on high strength, low alloy steel demonstrated an influence of steel cleanliness on plane strain fracture toughness K_{Ic}. Fig. 16 illustrates a variation of K_{Ic} from 20 to 14.3 kg-mm$^{1/2}$/mm^2 (56.8 to 40.5 ksi-in$^{1/2}$) for 4340 steel containing various quantities of inclusions. The quantity of inclusions present was determined with Quantimet (Quantitative television microscope), an image analysing computer based on an electronic scanning feature, the reliability of which was criticised previously [7, 14]. For 300 M steel (182 kg/mm^2 yield) the range of K_{Ic} for various inclusion levels was 18.3 to 16.4 kg-mm$^{1/2}$/mm^2 (52 to 46.5 ksi-in$^{1/2}$). However, since the variation obtained by these investigators is only on the order of 1.8 to 5.3 kg-mm$^{1/2}$/mm^2 (5 to 15 ksi-in$^{1/2}$), and the method of inclusion assessment has not proved to be consistent, the curve in fig. 16 is questionable.

5. Summary

Since most of the investigations reviewed herein have been conducted independently, the methods of inclusion assessment varied considerably. Despite this, the individual results generally indicated similar trends. Fatigue properties, transverse reduction of area, and impact strength were most seriously affected by inclusion contamination. Ultimate tensile strength gener-

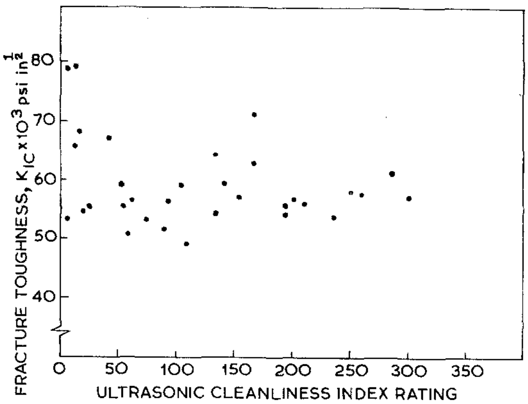

Figure 15 Fracture toughness versus material cleanliness [7].

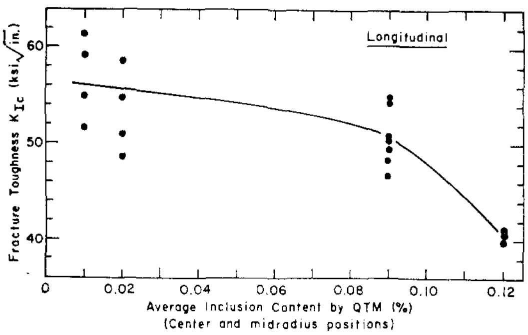

Figure 16 Fracture toughness versus inclusion content [29].

ally appears not to be influenced by nonmetallic inclusions. Fracture toughness also appears to be independent of inclusion contamination as measured ultrasonically, but only two quantitative studies have been reported concerning this parameter.

6. Conclusions

(i) In many applications, the type of inclusions are more important than the total inclusion content. This was evidenced consistently in the transverse ductility studies.

(ii) Although details of a relationship are vague, it is evident that as strength increases, the notch effect of inclusions becomes more significant.

(iii) Inclusions which are brittle relative to the steel, are more detrimental to the fatigue properties than inclusions which deform plastically.

(iv) It is possible that nonmetallic inclusions have an explicit effect on the low cycle fatigue properties of steel.

(v) Overall, the effects of inclusions on mechanical properties are governed by their size, shape, quantity, inter-spacing, distribution, orientation, interfacial strength, and physical properties relative to the matrix. Due to the numerous factors involved, any general correlation between nonmetallic inclusions and mechanical properties is highly improbable. While it is possible to obtain a plausible relationship in a particular situation, the factors involved must be standardised to such an extent that application of the results is extremely limited.

Acknowledgement

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Book Review

Quantitative Stereology

E. E. Underwood

Pp 274 (Addison-Wesley, 1970) £1·85.

The publication of this long-awaited book is an event of major importance for materials scientists who occasionally look down a microscope – that is, for materials scientists. It represents nothing less than a complete and systematic treatment of the quantitative aspects of microstructure. It is not only the best comprehensive account of the subject – it is also the only one. Most of the information collected here has appeared in fragmented form, but it has never before all been brought together in one place.

At first sight, the kind of problem with which Dr Underwood concerns himself is quite simple: for instance, he may want to know the grain size distribution of a piece of metal, the volume fraction and mean separation of a dispersed phase, the degree of orientation of dislocation lines, the degree of elongation of the grains in a recrystallised structure. It is not until one reads the painstakingly rigorous treatment of problems such as these, and the acute care taken in defining terms and proving relationships, that one comes to appreciate the complexity behind the seeming simplicity. As with $E = Mc^2$, the end-results are often simple but the arguments leading to them are not.

Essentially the book is concerned with the derivation of quantitative information about three-dimensional microstructure from measurements made on two-dimensional sections. The author begins by distinguishing the principal categories of measurements – point counts, lineal analysis, area summations – and assessing the basis on which statistical errors can be estimated. He goes on to derive by statistical methods the exact relationships between different one-, two- and three-dimensional quantities, such as for instance those between area and volume fractions of a phase. It is here that a number of surprisingly simple relationships emerge; the author is always punctilious about establishing their correctness by mathematical argument.

The book continues with the methods of quantifying degrees of orientation in microstructure, both for arrays of lines and for arrays of grains in two-dimensional sections. He continues with an analysis of particle and grain characteristics, initially those with uniform dimensions. This is particularly interesting in its detailed analysis of the concepts of diameter and mean free distance. The next chapter deals with the mathematially extremely difficult task of deriving a true *distribution* of grain or particles sizes from an apparently two-dimensional distribution. This is difficult, of course, because even if the grains are in fact all of the same size, the apparent sizes of their cuts by the plane of